

# Dynamically Induced Locking and Unlocking Transitions in Driven Layered Systems with Quenched Disorder

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Using numerical simulations, we examine a simple model of two or more coupled one-dimensional channels of driven particles with repulsive interactions in the presence of quenched disorder. We find that this model exhibits a remarkably rich variety of dynamical behavior as a function of the strength of the quenched disorder, coupling between channels, and external drive. For weaker disorder, the channels depin in a single step. For two channels we find dynamically induced decoupling transitions that result in coexisting pinned and moving phases as well as moving decoupled phases where particles in both channels move at different average velocities and slide past one another. Decoupling can also be induced by changing the relative strength of the disorder in neighboring channels. At higher drives, we observe a dynamical recoupling or locking transition into a state with no relative motion between the channels. This recoupling produces unusual velocity-force signatures, including negative differential conductivity. The depinning threshold shows distinct changes near the decoupling and coupling transitions and exhibits a peak effect phenomenon of the type that has been associated with transitions from elastic to plastic flow in other systems. We map several dynamic phase diagrams showing the coupling-decoupling transitions and the regions in which hysteresis occurs. We also examine the coexistence regime for channels with unequal amounts of quenched disorder. For multiple channels, multiple coupling and decoupling transitions can occur; however, many of the general features found for the two channel system are still present. Our results should be relevant to depinning in layered geometries in systems such as vortices in layered or nanostructured superconductors and Wigner or colloidal particles confined in nano-channels; they are also relevant to the general understating of plastic flow.

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## I. INTRODUCTION

A collection of interacting particles on an ordered or disordered substrate undergoes a depinning transition under an applied drive from a pinned state with immobile particles to an elastic or plastic sliding state [1–10]. In an elastic sliding state, each particle keeps the same neighbors over time [1–3], while in a plastic sliding state, the particles do not keep the same neighbors [4, 5, 10–14]. Many systems show a transition from elastic depinning for weak substrates to plastic depinning for strong substrates that is associated with changes in the velocity force curves and the depinning threshold as well as the onset of strong hysteretic effects [5, 11, 15, 16]. For example, the peak effect phenomenon for the transport of superconducting vortices, where a peak in the critical depinning force or critical current appears as a function of temperature or magnetic field, has been associated with a transition from elastic to plastic depinning and is known to be related to the disordering of the vortex lattice [5, 11]. Although the peak effect is primarily associated with superconductors, it can in principle appear in other systems that undergo a transition from elastic to plastic depinning. Several recent studies of colloidal systems and frictional systems have revealed peak effect-like behavior [9, 15, 16].

Plastic depinning transitions exhibit a wide variety of characteristics depending on whether the substrate is random [4] or periodic [6, 9, 17, 17–20]. The resulting plastic flow ranges from motion in winding channels to

avalanche behaviors with strong fluctuations, coexistence of large pinned and flowing regions, or transitions from two-dimensional (2D) mixing phases to one-dimensional (1D) decoupled channel phases. For higher drives, many systems undergo a transition from plastic flow to a more ordered flow state, such as a moving smectic state where the particles organize into 1D channels that can slide past one another [14, 15, 21] or partially ordered moving crystal states [6, 7, 13, 21].

Due to the complexity of plastic depinning phenomena in 2D and three-dimensional (3D) systems, numerous simpler systems have been proposed that still exhibit plastic depinning, such as particles flowing in a series of simple coupled layers. Many systems with depinning transitions can be modeled effectively as layered systems, including sliding charge density waves [1, 22] and vortices in strongly layered superconductors [23, 24]. Advances in fabrication techniques have made it possible to create nanostructured systems in which particles such as vortices move through effectively 1D coupled channels [25]. Marchetti *et al.* considered a mean field anisotropic slip model of coupled channels of particles oriented parallel to a driving force [26]. Along each channel, the particle interactions are elastic, but slip can occur between neighboring channels. In this model, when the depinning is elastic the channels show no slip or hysteresis, but for plastic depinning there is both slip and hysteresis. Further theoretical work on systems with only two layers demonstrated that a transition from non-hysteretic elastic depinning to hysteretic plastic depinning occurs

as a function of disorder [27]. For 3D layered sliding charge density wave systems, a transition from an elastic depinning to plastic hysteretic depinning has also been predicted [22], along with a second coupling transition at higher drives when the charge density waves begin to flow coherently. Numerical work on 3D layered superconducting vortex systems has shown a similar disorder-induced transition from elastic to plastic flow marked by a large increase in the depinning threshold when the layers decouple, which is typical of a peak effect behavior [23, 24, 28]. We note that although the transition to plastic depinning is marked by an increase in the depinning threshold, in real superconductors the depinning threshold decreases as the superconducting critical temperature or field is approached due to the changing penetration depth. This feature is generally not included in most simulations.

In this work we consider a simple depinning model consisting of two or more coupled 1D channels of repulsively interacting particles, all of which are subjected to a driving force and quenched disorder. This system could be experimentally realized using colloidal particles in coupled 1D channels with a disordered substrate and driven by an external electric field [29–32], coupled 1D wires containing Wigner crystal phases [33–36], or vortices on corrugated substrates driven parallel to the corrugation. Our simulations of this model include much more detail than can be captured in mean field studies [26, 27, 37]. In addition to confirming many of the predictions from the theoretical studies, we find a rich variety of new features. We induce decoupling transitions by changing the relative strength of the disorder from channel to channel, something that has not been considered in previous work on the depinning of layered systems. We also study commensuration effects by varying the particle density in one channel relative to the density in other channels. We drive all of the layers of particles; previous studies of this type of model considered transformer geometries with the drive applied to only one layer [38, 39]. Despite the apparent simplicity of our model, we find that even the two layer system has a wide variety of dynamical phases and exhibits all the salient features found for elastic and plastic depinning phenomena, including a peak effect at the transition between the two types of depinning. We also show that the transport signatures of the dynamical coupling and recoupling transitions can be enhanced in systems where the channels do not all have the same amount of disorder, suggesting that experimental realizations of this geometry may be an excellent way to probe these dynamical states.

The paper is organized as follows. In Section II we describe our model and simulation technique. Section III discusses decoupling transitions in a two channel system as the spacing between channels is varied, as well as a peak effect appearing at the transition from elastic to plastic depinning. In Section IV, we consider two channel samples in which the strength of the disorder differs in the two channels, where we observe drag-induced

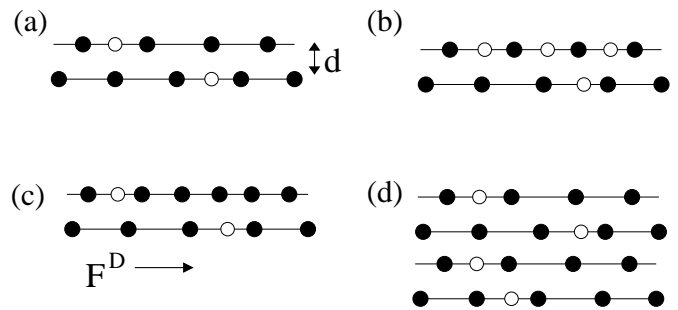


FIG. 1: Schematic of the system. The particles (filled circles) are constrained to move along 1D channels that are separated by a distance  $d$ . Each channel contains  $N_j$  particles as well as  $n_j$  randomly placed pinning sites (open circles) that each have a maximum pinning force of  $F_j^p$ . A uniform drive  $F^D$  is applied to all particles in the positive  $x$  direction. The particles interact via a repulsive Yukawa potential with other particles in the same channels and in neighboring channels. (a) A two channel system with  $N_1 = N_2$  and  $n_1 = n_2$ . (b) Two channels with an equal number of particles in each channel,  $N_1 = N_2$ , but with more pinning in the upper channel,  $n_1 > n_2$ . (c) Two channels with equal pinning in each channel,  $n_1 = n_2$ , but more particles in the upper channel,  $N_1 > N_2$ . (d) Multiple channels each containing the same number of particles,  $N_j = N_1$ , and pins,  $n_j = n_1$ .

pinning as well as hysteretic velocity-force signatures. In Section V, we find commensuration effects when the number of particles in each channel is varied for a two channel system, and we also study the effect of changing the particle-particle interaction prefactor. We introduce an eight channel system in Section VI and show that as disorder strength and channel coupling is varied, the behavior resembles that of the two channel system. We also study representative examples of eight channel systems in which not all channels contain the same number of particles. We conclude with a summary in Section VII.

## II. SIMULATION

We model an array of  $M$  coupled 1D channels separated by a distance  $d$  where channel  $j$  contains  $N_j$  particles [38]. The total number of particles in the system is  $N = \sum_{j=1}^M N_j$ . The particle-particle interactions are repulsive, and each particle interacts both with particles in the same channel as well as with particles in nearby channels. The number of randomly placed pinning sites in channel  $j$  is given by  $n_j$ . All of the particles in the system are subjected to a uniform external driving force  $F^D$ . We apply periodic boundary conditions in the  $x$  direction, along the length of the channels, and the total length of our system is  $L = 72a_0$ , where  $a_0$  is our unit of length that is typically a micron in colloidal systems. The boundaries are open along the  $y$  direction transverse to the channels. Fig. 1 shows a schematic of our sys-

tem in several different configurations. In Fig. 1(a) there are  $M = 2$  channels with equal numbers of particles,  $N_1 = N_2$ , and pins,  $n_1 = n_2$ , in each channel. Figure 1(b) shows a two channel system with equal numbers of particles but with more pinning sites in the upper channel,  $n_1 > n_2$ , while in Fig. 1(c) there are equal numbers of pins but more particles in the upper channel,  $N_1 > N_2$ . Figure 1(d) shows a multiple channel  $M = 5$  system with equal numbers of particles in each channel,  $N_j = N_1$ , and equal numbers of pins in each channel,  $n_j = n_1$ .

The particles interact via a Yukawa or screened Coulomb potential that is appropriate for charged colloids or for charge transport in the presence of screening. We show later that the same general dynamic phases also occur for repulsively interacting vortices in type-II superconductors. For the colloidal system, the dynamics of a particle  $i$  is determined by integrating the following overdamped equation of motion:

$$\eta \frac{d\mathbf{R}_i}{dt} = \mathbf{F}_i^{pp} + \mathbf{F}_i^s + \mathbf{F}^D + \mathbf{F}_i^T. \quad (1)$$

Here  $\mathbf{R}_i$  is the location of particle  $i$  and  $\eta$  is the damping coefficient that is set equal to one. The particle-particle interaction force is  $\mathbf{F}_i^{pp} = -\sum_{j \neq i}^N \nabla V(R_{ij}) \hat{\mathbf{R}}_{ij}$  where  $V(R_{ij}) = (q^2 E_0 / R_{ij}) \exp(-\kappa R_{ij})$ ,  $R_{ij} = |\mathbf{R}_i - \mathbf{R}_j|$ ,  $\hat{\mathbf{R}}_{ij} = (\mathbf{R}_i - \mathbf{R}_j) / R_{ij}$ ,  $E_0 = Z^{*2} / 4\pi\epsilon\epsilon_0$ ,  $\epsilon$  is the solvent dielectric constant,  $Z^*$  is the dimensionless squared colloid charge that is taken to be  $q^2 = 2.0$  unless otherwise noted. We take the screening length  $1/\kappa = 4a_0$  and choose the distance between channels in the range  $d \leq 3a_0$  (equivalent to  $d/\kappa \leq 2$ ) to ensure that particles in neighboring channels interact with each other. In the absence of pinning, the particles in an isolated channel would adopt a lattice spacing of  $a_j = N_j / L$ . The substrate pinning force is given by  $\mathbf{F}_i^s = \sum_{k=1}^{n_j} F_j^p (R_{ik}^p / R_p) \Theta(R_p - R_{ik}^p) \hat{\mathbf{R}}_{ij}^p$ . Here particle  $i$  sits in channel  $j$ ,  $n_j$  is the number of pinning sites in channel  $j$ ,  $\Theta$  is the Heaviside step function,  $\mathbf{R}_k^p$  is the location of pinning site  $k$ ,  $R_{ik}^p = |\mathbf{R}_i - \mathbf{R}_k^p|$ , and  $\hat{\mathbf{R}}_{ik}^p = (\mathbf{R}_i - \mathbf{R}_k^p) / R_{ik}^p$ . The pinning site radius is  $R_p$  and the maximum strength of the pinning sites in channel  $j$  is  $F_j^p$ . Within each channel the pins are placed in randomly chosen non-overlapping positions that are different for each channel. This pinning model has been used in previous simulations to represent quenched disorder for colloidal systems, classical electron systems, and vortices in superconductors. The external drive  $\mathbf{F}^D = F^D \hat{\mathbf{x}}$  is applied in the positive  $x$ -direction. The drive is increased in increments of  $\delta F^D = 0.001$ , and after each increment  $F^D$  is held fixed for  $10^5$  simulation time steps to ensure that any transient effects have subsided. The thermal force  $\mathbf{F}_i^T$  is modeled by adding a Langevin noise term with the following properties:  $\langle F_i^T(t) \rangle = 0$  and  $\langle F_i^T(t) F_j^T(t') \rangle = 2\eta k_B \delta_{ij} \delta(t - t')$ , where  $k_B$  is the Boltzmann constant. We initially consider the  $T = 0$  case. We measure the average particle velocity in each channel for every force increment,  $V_j = \langle N_j^{-1} \sum_{i=1}^{N_j} v_i \rangle$ , where

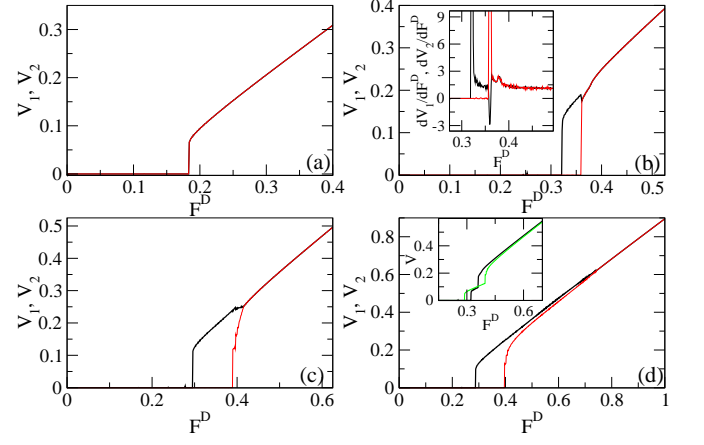


FIG. 2: The velocity of each channel,  $V_1$  (dark line) and  $V_2$  (light red line), in a two channel ( $M = 2$ ) system versus applied drive  $F^D$  with fixed quenched disorder for varied distance  $d$  between the channels. Each channel has the same number of particles,  $N_1 = N_2$ , and pinning sites  $n_1 = n_2$ . Here  $n_1/N_1 = 0.25$ ,  $F_1^p = F_2^p = 6.0$ ,  $R_p = 0.23a_0$ , and the in-channel lattice constant  $a_1 = a_2 = L/N_1 = 1.5a_0$ . (a) For  $d/a_1 = 1.133$ , there is an elastic depinning transition and both channels depin simultaneously. (b) At  $d/a_1 = 1.47$ , channel 1 depins first while channel 2 remains pinned. Once channel 2 depins, the channels become dynamically locked. The locking transition is associated with negative differential conductivity (NDC) in  $V_1$ , as illustrated in the inset where  $dV_1/dF^D < 0.0$  at  $F^D = 0.36$ . (c) For  $d/a_1 = 1.57$  near  $F^D = 0.39$  there is a region where both channels are depinned but  $V_1 \neq V_2$ , indicating that the channels are sliding past one another. Dynamical locking occurs for  $F^D > 0.415$ . (d) At  $d/a_1 = 1.64$ , the recoupling transition is shifted to much higher  $F^D$ . Inset: The sum of the velocities  $V = V_1 + V_2$  for  $d/a_1 = 1.47$  (dark line) and  $d/a_1 = 1.64$  (light green line) shows the occurrence of two step depinning; there is no NDC in  $V$  for the  $d/a_1 = 1.47$  sample.

$v_i = (d\mathbf{R}_i/dt) \cdot \hat{\mathbf{x}}$ . We also measure the net velocity of all the channels,  $V = \sum_{i=1}^M V_i$ .

### III. DECOUPLING TRANSITIONS FOR TWO LAYER SYSTEMS

We first consider a two layer system in which we vary the interlayer distance  $d$ . We can effectively decrease the coupling between the layers by increasing  $d$  since the interaction between Yukawa particles becomes weaker for increasing interparticle distance. In Fig. 2 we plot the velocity force curves for channels 1 and 2 in a system with equal numbers of particles in each channel ( $N_1 = N_2$ ), in-channel lattice constant  $a_1 = a_2 = 1.5a_0$ , equal numbers of pins in each channel ( $n_1 = n_2$ ),  $n_1/N_1 = 0.25$ ,  $F_1^p = F_2^p = 6.0$ , and  $R_p = 0.23$  for varied  $d$  measured in terms of  $d/a_1$ . For small  $d$  or strong coupling, the two channels depin simultaneously without any slipping between the channels, as shown in Fig. 2(a) for  $d/a_1 = 1.333$ . All the

particles keep the same neighbors, indicating that the depinning is elastic. The value of  $F^D$  at which depinning first occurs is termed the critical depinning threshold,  $F_c$ .

As  $d$  increases there is a transition to a state where the channels depin individually, as shown in Fig. 2(b) for  $d/a_1 = 1.47$ . Here channel 1 depins at  $F^D = 0.32$  while channel 2 remains pinned until  $F^D = 0.36$ . Since both channels have equal numbers of pins, equal numbers of particles, and experience the same driving force, the difference in depinning threshold arises due to the random placement of the pinning sites in the two channels, which differs from channel to channel. As soon as channel 2 depins, the system transitions directly into a state where the moving channels are dynamically locked with each other and move at the same velocity,  $V_1 = V_2$ , without any slipping. As the inset in Fig. 2(b) shows, the dynamic locking transition coincides with a drop in  $V_1$  indicative of *negative* differential conductivity (NDC), where a system of particles shows a decrease in the velocity under increasing drive,  $dV_1/dF^D < 0.0$ . The NDC occurs only in channel 1 at  $F^D = 0.36$  when  $V_2$  jumps from zero to a finite value, which also corresponds to a peak in  $dV_2/dF^D$ .

Fig. 2(c) shows the velocity force curves for  $d/a_1 = 1.57$ , where the depinning threshold for channel 1 is again lower than that for channel 2. In this case, after channel 2 depins the system does not immediately enter the moving locked phase. Instead, we find a region  $0.39 < F^D < 0.415$  with  $V_1 \neq V_2$ ,  $V_1 > 0$ , and  $V_2 > 0$ . Here both channels are moving but at different average velocities, indicating that the particles in channel 1 are sliding past the particles in channel 2. The channels become dynamically locked for  $F^D > 0.415$ , where  $V_1 = V_2$ . As  $d/a_1$  is further increased, the dynamical locking transition shifts to larger values of  $F^D$ . For example, in Fig. 2(d) for  $d/a_1 = 1.64$ , the channels are only dynamically coupled for  $F_D > 0.742$ .

In the inset of Fig. 2(d) we plot the overall velocity  $V = V_1 + V_2$  versus  $F^D$  for the samples with  $d/a_1 = 1.47$  and  $d/a_1 = 1.64$  from Fig. 2(b) and Fig. 2(d), respectively. These are the curves that would be obtained in an experimental measurement of net transport rather than individual channel transport. For  $d/a_1 = 1.47$ , where NDC of  $V_1$  appeared in Fig. 2(b), the inset of Fig. 2(d) shows that there is no NDC in the overall velocity  $V$ . This is because the decrease in  $V_1$  at  $F^D = 0.36$  is exactly compensated by the increase in  $V_2$ , so there is no net drop in  $V$  at the second depinning transition. For both  $d/a_1 = 1.47$  and  $d/a_1 = 1.64$ ,  $V$  shows a characteristic step feature that indicates the presence of a two-step depinning transition. On the lower step in  $V$ , only channel 1 is depinned, while on the upper step, both channels are flowing. For this particular set of parameters, we find only a weak signature in  $V$  of the relocking transition that occurs at higher drives; however, for other parameters, we will show below that the onset of dynamical locking produces much more pronounced effects in  $V$ .

By conducting a series of simulations we map the dy-

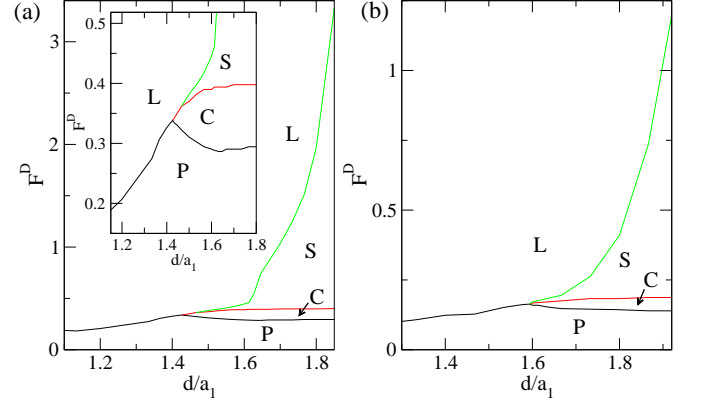


FIG. 3: Dynamic phase diagrams  $F^D$  vs  $d/a_1$  for the  $M = 2$  system from Fig. 2 with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.23a_0$ , and  $a_1 = a_2 = 1.5a_0$ . P: pinned phase with  $V_1 = V_2 = 0$ ; C: coexistence phase where one channel is moving but the other is pinned; S: sliding phase where both channels are moving but  $V_1 \neq V_2$ ; L: dynamically locked phase where  $V_1 = V_2 > 0$ . (a) The sample from Fig. 2 with  $F_1^p = F_2^p = 6.0$ . For  $d/a_1 < 1.43$  the system depins directly into the locked phase, while for  $d/a_1 > 1.43$ , dynamically induced locking occurs at a value of  $F^D$  that increases with increasing  $d/a_1$ . Inset: A blowup of the main panel illustrating that the P phase reaches its maximum value of  $F^D$  at the transition from elastic to plastic depinning. (b) A sample with  $F_1^p = F_2^p = 3.0$  has the same features but the onsets of the S and C phases fall at higher values of  $d/a_1 > 1.6$ .

namical phase diagram for  $F^D$  vs  $d/a_1$  for the sample from Fig. 2 with  $F_1^p = F_2^p = 6.0$  in Fig. 3(a) and for a sample with  $F_1^p = F_2^p = 3.0$  in Fig. 3(b). In the pinned (P) phase, both channels are pinned,  $V_1 = V_2 = 0$ . In the coexistence (C) phase, one channel is pinned while the other is moving. The sliding (S) phase has both channels moving but not locked,  $V_1 > 0$  and  $V_2 > 0$  but  $V_1 \neq V_2$ , while in the locked (L) phase both channels move together without slipping,  $V_1 = V_2 > 0$ . In Fig. 3(a), for  $d/a_1 > 1.43$  the layers decouple at depinning and the width of region S grows with increasing  $d/a_1$ . The width of the coexistence phase saturates as  $d/a_1$  increases due to the decoupling of the particles, which causes the depinning thresholds of each channel that mark the borders of the C phase to be determined only by the quenched disorder configuration and the interactions among particles within the channel, and to be insensitive to the positions of the particles in the neighboring channel. The recoupling that marks the transition between the S and L phases rapidly shifts to higher  $F^D$  with increasing  $d/a_1$  for  $d/a_1 > 1.613$ . In Fig. 3(b) we show that in a sample with weaker pinning of  $F_1^p = F_2^p = 3.0$ , the transition directly from P to L persists out to the higher value of  $d/a_1 = 1.6$  but that the same general features found for higher  $F^p$  in Fig. 3(a) still occur and are shifted to higher  $d/a_1$  and lower  $F^D$ .

The dynamical phase diagrams in Fig. 3 have many similarities to the dynamical phase diagrams obtained for

2D and 3D driven vortex systems [14, 15, 24, 26, 37]. The 2D vortex system exhibits pinned, plastic, and dynamically ordered phases, with the vortex lattice ordering into either a moving smectic or moving crystal configuration [13–15]. Our system has the same pinned phase, while the coexistence phase that contains a mixture of moving and pinned channels corresponds to the plastic flow regime for the vortex system. The sliding phase we observe resembles the moving smectic phase found for vortices, while our locked phase is equivalent to the 2D dynamically reordered phase or, for 3D vortex systems, to the transition from decoupled layers to coupled 3D vortex lines [23, 28, 40]. In the vortex system, as the pinning strength increases or the vortex-vortex interactions are weakened, the reordering transition shifts to higher drives [14]. The dynamical reordering occurs because the pinning is effectively weakened when the vortices are moving rapidly. In our system, in the locked phase the effectiveness of the quenched disorder is dramatically reduced. As the interchannel particle-particle interaction strength decreases for increasing  $d/a_1$ , the quenched disorder becomes more effective, the recoupling transition shifts to higher  $F^D$ , and the additional C and S phases appear between the P and L phases.

In the inset of Fig. 3(a) we plot a blowup of the region near  $d/a_1 = 1.4$  for the  $F_1^p = F_2^p = 6.0$  sample where the transition between elastic depinning directly into the locked phase and plastic depinning into the coexistence phase occurs. The critical depinning threshold  $F_c$  reaches a peak at  $d/a_1 = 1.43$  at the transition, while for  $d/a_1 > 1.43$ ,  $F_c$  decreases and then saturates with increasing  $d/a_1$ . The  $F_c$  peak has all the hallmark features of the peak effect phenomenon found for vortices and other systems where a peak in the critical depinning threshold occurs near the transition from elastic to plastic depinning [5, 11, 15, 16]. In many of these studies the  $F_c$  peak occurs inside the plastic depinning regime just beyond the transition point, while  $F_c$  for the elastic depinning is always lower than for the plastic depinning [5]. In our two layer system, the  $F_c$  peak falls right at the transition out of the elastic depinning regime, and  $F_c$  decreases with increasing  $d/a_1$  within the plastic depinning regime.

Determining whether the effectiveness of the pinning in the plastic or decoupled phases is higher than in the elastic phase can be ambiguous when different quantities are measured [5, 11]. If we consider only the overall velocity  $V$  at a fixed value of  $F^D$  as we vary  $d/a_1$ , we find that  $V$  drops in the decoupled regime, suggesting that the pinning is more effective in the decoupled state. This contrasts with the behavior of the depinning threshold  $F_c$ , which drops in the decoupled regime, suggesting that the pinning is less effective in the decoupled state. In Fig. 4, we plot  $V$  versus  $d/a_1$  for the sample from Fig. 3(a) with  $F_1^p = F_2^p = 6.0$  for fixed  $F^D$  values ranging from  $F^D = 0.4$  to  $F^D = 1.0$ . For  $F^D \leq 0.8$ ,  $V$  drops or becomes zero at the decoupling transition, and for  $0.6 \leq F^D \leq 0.8$  the value of  $V$  for  $d/a_1$  above the de-

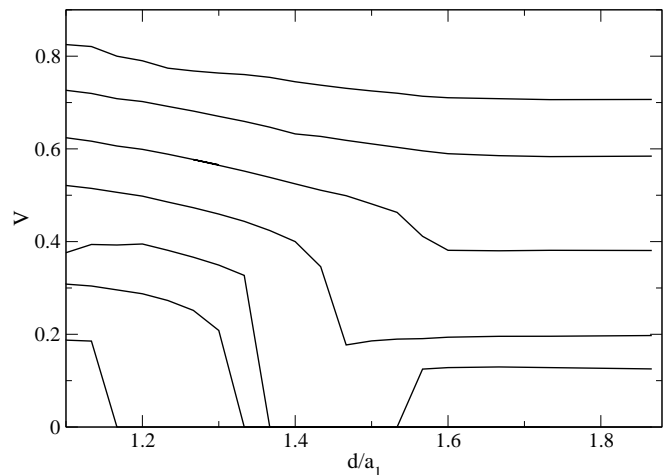


FIG. 4: The summed velocity  $V = V_1 + V_2$  for the  $M = 2$  system in Fig. 3(a) with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.23a_0$ ,  $a_1 = 1.5a_0$ , and  $F_1^p = F_2^p = 6.0$  at  $F^D = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ , and  $1.0$ , from bottom to top. The value of  $V$  in the decoupled regime is always lower than in the coupled regime, and the dip in  $V$  at the decoupling transition gradually fades away as  $F^D$  increases.

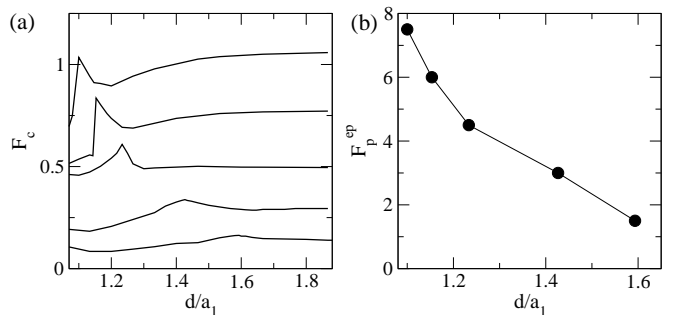


FIG. 5: (a) The critical depinning threshold  $F_c$  vs  $d/a_1$  for the  $M = 2$  system in Fig. 3(a) with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.23a_0$ , and  $a_1 = 1.5a_0$  for varied  $F_1^p = F_2^p = 15.0, 12.0, 9.0, 6.0$ , and  $3.0$ , from top to bottom. In each case the peak in  $F_c$  falls at the transition from elastic to plastic depinning. (b)  $F_p^{ep}$ , the value of  $F_1^p$  and  $F_2^p$  at which a transition from elastic to plastic depinning occurs, vs  $d/a_1$  in the same system.

coupling transition is smaller than the value of  $V$  for  $d/a_1$  below the decoupling transition. For  $F^D > 0.8$ , the size of the dip in  $V$  at the decoupling transition gradually diminishes. This behavior is very similar to that observed across the peak effect for vortices in type-II superconductors, and indicates that for the moving particles, the pinning is more effective in the decoupled phase than in the coupled phase. The behavior of  $V$  differs from that of  $F_c$  because  $V$  is associated with moving particles while  $F_c$  is determined by the static configuration of the pinned particles.

In Fig. 5(a) we plot  $F_c$  versus  $d/a_1$  for a range of  $F_p$  values in the system from Fig. 3(a) to illustrate that the



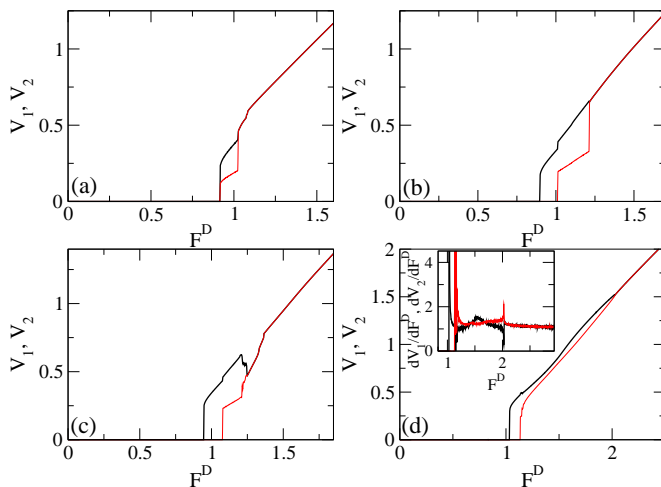


FIG. 6:  $V_1$  (dark line) and  $V_2$  (light red line) vs  $F^D$  for the  $M = 2$  system in Fig. 5(a) with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.23a_0$ ,  $a_1 = 1.5a_0$ , and  $F_1^p = F_2^p = 15.0$  for (a)  $d/a_1 = 1.147$ , (b)  $d/a_1 = 1.2$ , (c)  $d/a_1 = 1.27$ , and (d)  $d/a_1 = 1.47$ . Inset of (d): The  $dV_1/dF^D$  and  $dV_2/dF^D$  vs  $F^D$  curves both peak at the dynamical coupling transition at  $F^D = 2.0$ ; for  $F^D > 2.0$  above the transition, the fluctuations in both curves show increased correlation.

decoupling transition is associated with a peak in  $F_c$ . On average,  $F_c$  increases with increasing  $F^p$  and with increasing  $d/a_1$ . The decoupling transition, and with it the peak in  $F_c$ , shifts to lower values of  $d/a_1$  with increasing  $F^p$  and becomes more prominent. In all cases the peak in  $F_c$  falls exactly at the transition from elastic to plastic flow. For a given value of  $d/a_1$ , it is possible to induce a decoupling transition by increasing  $F^p$ ; the value of  $F_1^p$  and  $F_2^p$  at the decoupling transition is termed  $F_p^{ep}$ . In Fig. 5(b) we plot  $F_p^{ep}$  versus  $d/a_1$ , where we find that as  $d/a_1$  decreases and the coupling between the channels increases,  $F_p^{ep}$  increases faster than linearly.

The velocity force signatures illustrated in Fig. 2 are generally robust against changes in pinning strength. For samples with stronger pinning,  $F_1^p = F_2^p > 6.0$ , many of the features become more prominent. For sufficiently strong  $F^p$ , the system can depin directly into the sliding state with no intermediate coexistence state, as illustrated in 6(a) for  $F_1^p = F_2^p = 15.0$  and  $d/a_1 = 1.147$ . Here both channels depin simultaneously at  $F^D = 0.915$ , and for  $0.915 < F^D < 1.025$  the channels move at different velocities with  $V_1 \neq V_2$ . We also find a larger number of jumps in the velocity force curves for higher  $F^p$ , as shown in Fig. 6(a) near  $F^D = 1.08$ . The jumps are indicative of the solitonlike nature of the particle motion in channels with strong pinning below the recoupling transition. Due to the random spatial placement of the pinning sites, the individual pins vary in their effectiveness at trapping particles. As the driving force is increased below depinning, particles escape from the least effective pins only to pile up behind particles that are

trapped in the more effective pins. Since the channel is 1D, flowing particles cannot pass trapped particles. As the driving force further increases, particles in the pileup regions approach each other more closely and exert a greater force on the trapped particle that produced the pileup. At depinning, the trapped particle escapes from the pin and is immediately replaced by another particle that becomes pinned in its place. The trapped particle travels across the system and rejoins the pileup; the extra force it exerts on the particles ahead of it causes the trapped particle at the front of the pileup to depin and repeat the process. This picture is oversimplified; in actuality, each channel contains multiple trapping sites that create pileups and particles jump from one pileup to another above depinning. Each of the trapping sites is associated with some local depinning threshold  $F_c^{loc}$ . As  $F^D$  increases, one trapping site after another reaches the condition  $F_c^{loc} < F^D$  and ceases to trap particles. The resulting enhancement of the mobility of all the particles in the channel manifests itself as a jump in channel velocity  $V_j$ . As  $F^p$  increases, there is a greater spread in  $F_c^{loc}$ , producing a larger number of steps in  $V_1$  and  $V_2$  above depinning. We note that such velocity force jumps are often called switching events in sliding charge density wave systems [1]. For vortex systems, near the peak effect regime a series of jumps and dips in the current-voltage curves and  $dV/dI$  curves can appear. These have been termed a fingerprint phenomenon since the same features repeat upon cycling the IV curves, indicating that the features result from the detailed configuration of the pinning sites [41, 42]. The curves in Fig. 6(a) are obtained for a specific pinning realization. For a different pinning realization, the general features of the velocity force curves remain the same but the location and height of the velocity jumps will change. In Fig. 6(b), a sample with the same disorder configuration and strength but with  $d/a_1 = 1.2$  has separate depinning thresholds for channels 1 and 2 along with a sharp transition into the locked phase at  $F^D = 1.2$ . In the  $d/a_1 = 1.27$  sample of Fig. 6(c), we observe a strong negative differential conductivity signature in  $V_1$  at the transition between the sliding and locked phases. There is also a change in the rate at which  $V_1$  and  $V_2$  increase with  $F^D$  in the locked phase, with a more rapid increase for  $1.25 < F^D < 1.365$  and a slower increase for  $F^D > 1.365$ . For  $d/a_1 = 1.47$  in Fig. 6(d), the channels do not lock until  $F^D = 2.03$ , and  $V_1$  shows additional structure within the sliding phase. For increasing  $F^p$  beyond what we show here, we find similar features in the velocity force curves; however, the number of jumps and anomalies in the curves further increases as the microscopic configurations of the pinning sites begin to dominate the behavior completely.

We next compare the velocity fluctuations in the individual channels in the unlocked and locked state. Fig. 7 shows the instantaneous velocities  $V_1(t)$  and  $V_2(t)$  for channels 1 and 2 as a function of time measured in simulation steps for the system in Fig. 2(c) at  $d/a_1 = 1.57$  with  $F_1^p = F_2^p = 6.0$ . At  $F^D = 1.395$  in Fig. 7(a), the

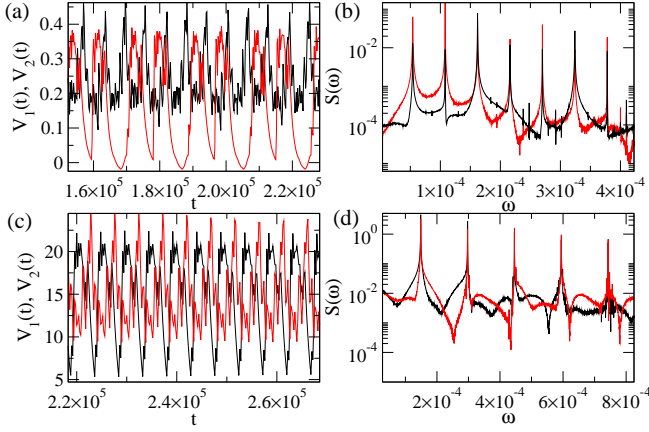


FIG. 7: (a) The instantaneous velocity of each channel  $V_1(t)$  and  $V_2(t)$  versus time in simulation steps for channels 1 (dark black line) and 2 (light red line) for the  $M = 2$  system in Fig. 2(c) with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.23a_0$ ,  $a_1 = 1.5a_0$ ,  $d/a_1 = 1.57$ , and  $F_1^p = F_2^p = 6.0$  at  $F^D = 0.395$ . The system is in the S phase and both channels are moving at different velocities. (b)  $S(\omega)$  for the time series in panel (a) shows that channel 2 has greater spectral power at the lowest frequencies compared to channel 1.  $\omega$  is reported in inverse simulation time steps. (c)  $V_1(t)$  and  $V_2(t)$  for the same system with  $F^D = 0.47$  where the channels are locked. (d)  $S(\omega)$  for the time series in panel (c) shows nearly equal spectral weight in each channel at the fundamental frequencies.

system is in the S phase and both channels are moving with different average velocities  $V_1 > V_2 > 0$ . Both channels show a periodic response but channel 1 has a higher frequency and higher average velocity, while  $V_2(t)$  drops nearly to zero during each cycle. In Fig. 7(b) we plot the corresponding power spectra  $S(\omega)$  of the velocity signals, with  $\omega$  measured in inverse simulation time steps. Channel 2 has greater spectral weight at the low frequency of  $\omega = 1.1 \times 10^{-4}$  corresponding to its fundamental frequency, while the peak at this frequency for channel 1 is orders of magnitude smaller in height. Channel 1 shows a response at this frequency due to its coupling with channel 2. Near  $\omega = 1.62 \times 10^{-4}$ , the fundamental frequency of the faster moving channel 1, the peak in  $S(\omega)$  is higher for channel 1 than for channel 2. This result shows that the two channels each have a periodic velocity signal corresponding to a different washboard frequency; however, since the channels are interacting, both velocity signals contain both washboard frequencies. In Fig. 7(c) we plot  $V_1(t)$  and  $V_2(t)$  for the same system at  $F^D = 0.47$  in the locked regime where the channels have the same average velocity. Here both channels exhibit the same fundamental frequency even though the shapes of  $V_1(t)$  and  $V_2(t)$  do not completely overlap. The corresponding  $S(\omega)$  in Fig. 7(d) shows that both channels have nearly the same spectral weight at the fundamental frequency of  $\omega = 1.48 \times 10^{-4}$  and its harmonics, unlike the unequal peaks which appeared in the unlocked phase in Fig. 7(b).

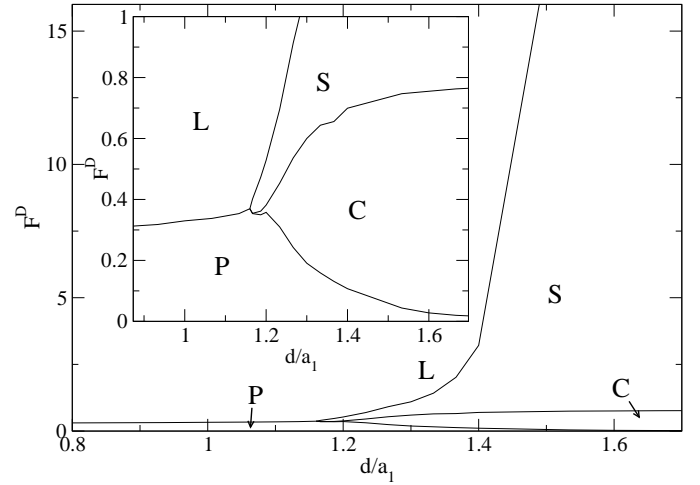


FIG. 8: The dynamic phase diagram  $F^D$  vs  $d/a_1$  for the  $M = 2$  system in Fig. 2 with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.23a_0$ ,  $a_1 = 1.5a_0$ ,  $F_1^p = 0$  and  $F_2^p = 15.0$ . P: pinned phase; C: coexistence phase where only one channel is moving; S: sliding phase where  $V_1 \neq V_2 > 0$ ; L: locked phase. As  $d/a_1$  increases, the depinning threshold for channel 1 goes to zero and the system immediately enters region C for small nonzero  $F^D$ . Inset: A blowup of the main panel near the transition from elastic to plastic depinning.

#### IV. VARIED RELATIVE DISORDER STRENGTH AND HYSTERESIS

We next consider the effects of varying the pinning strength from channel to channel in order to create one channel with high pinning strength and one channel with low pinning strength. Figure 8(a) shows the dynamic phase diagram of  $F^D$  versus  $d/a_1$  for a two channel system with  $F_1^p = 15.0$  and  $F_2^p = 0$ , while the inset illustrates a blowup of the area near the onset of plastic depinning. The strongly pinned particles in channel 1 are able to effectively pin the particles in channel 2 via particle-particle interactions alone, allowing the pinned regime to persist for finite values of  $F^D$  in spite of the fact that the pins in channel 2 have zero strength. For low  $d/a_1$  when the interactions between particles in neighboring channels are strong, the depinning of both channels occurs simultaneously in a single step at  $F_c$  from the pinned state to the moving locked state. As  $d/a_1$  increases,  $F_c$  increases and reaches a maximum value at  $d/a_1 = 1.16$  corresponding to the decoupling transition. For  $d/a_1$  above the decoupling transition, the depinning force  $F_c$  for channel 2 decreases monotonically with increasing  $d/a_1$ , unlike the saturation that occurred in Fig. 3 for samples with  $F_1^p = F_2^p$ . As  $d/a_1$  increases, the coupling between the channels decreases monotonically, and since the particles in channel 2 are pinned only due to their interactions with the pinned particles in channel 1,  $F_c$  for channel 2 decreases with increasing  $d/a_1$ . The transition line between the C and S phases, which corresponds to the depinning force  $F_c$  for channel

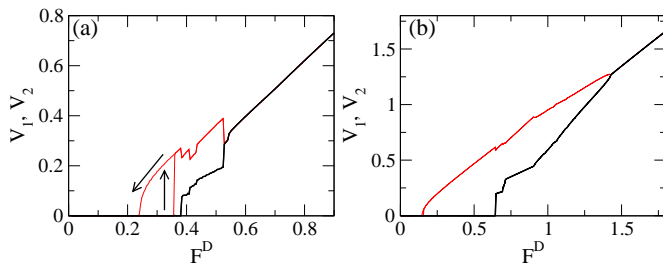


FIG. 9:  $V_1$  (dark line) and  $V_2$  (light red line) vs  $F^D$  under a cycled drive for the  $M = 2$  system in Fig. 8 with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.23a_0$ ,  $a_1 = 1.5a_0$ ,  $F_1^p = 15.0$  and  $F_2^p = 0.0$ . (a) At  $d/a_1 = 1.27$  we find hysteresis in  $V_1$  near the transition to the pinned phase but there is no hysteresis in  $V_2$ . The arrows indicate the  $V_2$  curves obtained for sweeping  $F^D$  up and down. (b) For the weaker coupling of  $d/a_1 = 1.33$ , there is no hysteresis and dynamic coupling occurs at  $F^D = 1.43$ .

1, grows more rapidly above the decoupling transition with increasing  $d/a_1$  in Fig. 8 than it did for samples with  $F_1^p = F_2^p$  in Fig. 3. This is because the particles in channel 2 produce relatively little effective drag for the particles in channel 1 when there is no pinning in channel 2, and this drag rapidly becomes nearly zero as  $d/a_1$  increases and the coupling between the channels decreases. The transition from the S to L phases also grows more rapidly with increasing  $d/a_1$  in Fig. 8 than for the samples with  $F_1^p = F_2^p$ . Our results show that coupling-decoupling transitions can persist even when the pinning in one channel is completely absent.

For the parameters examined so far in this work we observe only weak hysteresis in the velocity-force curves occurring near the sharp jumps in  $V_1$  or  $V_2$ . Hysteresis is often absent in 1D systems, as shown with the no-passing rule [43]; however, in studies using phase-field modeling, hysteresis has been observed in 1D systems [6]. In our system we never find hysteresis when the depinning is elastic, but hysteresis can occur for plastic depinning, as illustrated in Fig. 9(a), for a system with the same parameters as in Fig. 8 at  $d/a_1 = 1.27$ . Here hysteresis occurs at the depinning transition only for the pin-free channel 2 but not for channel 1. The summed velocity  $V = V_1 + V_2$  also exhibits hysteresis at depinning, but by examining the separate velocity signals we can determine that the hysteresis originates from only one of the channels. Figure 9(a) also shows that there is no hysteresis across the S-L transition at  $F^D = 0.526$  in spite of the discontinuous jump in both  $V_1$  and  $V_2$  that occurs at this transition. In Fig. 9(b) we plot  $V_1$  and  $V_2$  for cycled  $F^D$  in a more weakly coupled sample with  $d/a_1 = 1.33$ , further from the value of  $d/a_1 = 1.16$  where the transition from elastic to plastic depinning occurs. In this case we find dynamic locking of the channels above  $F^D = 1.43$ ; however, there is no hysteresis across any of the transitions.

We can also examine the effects of varying the relative

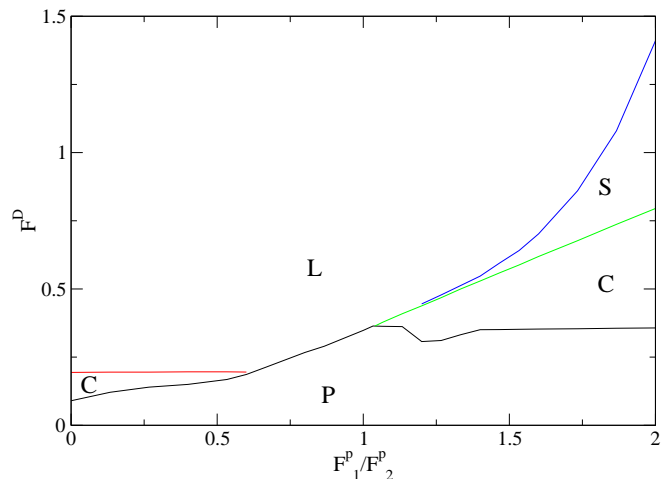


FIG. 10: Dynamic phase diagram  $F^D$  vs  $F_1^p/F_2^p$  for an  $M = 2$  system with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.182$ ,  $R_p = 0.35a_0$ ,  $a_1 = 1.5a_0$ ,  $d/a_1 = 1.7$ , and fixed  $F_2^p = 0.75$ . P: pinned phase; C: coexistence phase; S: sliding phase; L: locked phase. Near  $F_1^p/F_2^p = 1.0$ , where the pinning strength is the same in both channels, the system depins elastically. The depinning becomes plastic once  $F_1^p/F_2^p$  moves sufficiently far away from 1 in either direction.

pinning strength in the two channels by altering  $F_1^p/F_2^p$  for a system with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.182$ ,  $R_p = 0.35a_0$ , and  $d/a_1 = 1.7$  as shown in Fig. 10. Here  $F_2^p = 0.75$  and  $F_1^p$  is varied. For this particular set of parameters at  $F_1^p/F_2^p = 1.0$  the depinning is elastic. For  $0 < F_1^p/F_2^p < 0.6$ , the system depins plastically into the coexistence state where channel 1 is moving and channel 2 is pinned, and for higher  $F^D$  the system enters the locking regime without passing through the sliding phase. At  $F_1^p/F_2^p = 0$  the depinning threshold of channel 1 is nonzero due to the strong interaction of the particles in channel 1 with the pinned particles in channel 2. As  $F_1^p$  increases from zero the width of the pinned region grows as the depinning threshold of channel 1 rises; however, for  $0 < F_1^p/F_2^p < 0.6$  the depinning threshold of channel 2 remains constant since  $F_2^p$  is fixed. For  $0.6 < F_1^p/F_2^p < 1.0$  the depinning is elastic and both channels depin simultaneously; further, within this regime the pinned region has the most rapid growth for increasing  $F_1^p/F_2^p$ . For  $F_1^p/F_2^p > 1.0$  the depinning becomes plastic again and each channel depins separately; however, channel 2 now depins first since  $F_1^p > F_2^p$ . As  $F_1^p/F_2^p$  increases above 1, the width of the pinned phase passes through a small dip but remains nearly constant since the depinning threshold of channel 2 is determined by  $F_2^p$  which is held fixed. Near  $F_1^p/F_2^p = 1.2$  we find the onset of the sliding phase, which grows rapidly in width with increasing  $F_1^p/F_2^p$ . The sliding phase is followed at higher  $F^D$  by the dynamically locked phase. If we increase the fixed value of  $F_2^p$  and sweep  $F_1^p/F_2^p$ , the extent of the elastic depinning window diminishes as  $F_2^p$  becomes larger until for sufficiently large  $F_2^p$  there is no longer an elastic depin-



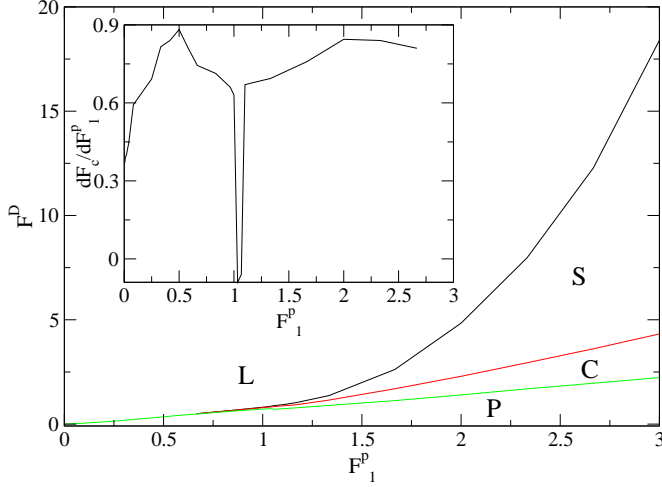


FIG. 11: Dynamic phase diagram  $F^D$  vs  $F_1^p$  for an  $M = 2$  sample with  $F_2^p = F_1^p$ ,  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.23a_0$ ,  $a_1 = 1.5a_0$ , and  $d/a_1 = 1.7$ . P: pinned phase; C: coexistence phase; S: sliding phase; L: locked phase. The depinning is elastic for  $F_p < 0.8$ . Inset:  $dF_c/dF_1^p$  for the same system shows a pronounced dip near the transition from elastic to plastic depinning.

ning regime. Instead, the sliding phase appears for all values of  $F_1^p/F_2^p$ . We have also considered samples with  $F_1^p = F_2^p$  but with different pinning densities  $n_1 \neq n_2$ , and we find the same general features as a function of  $n_1/n_2$  as we have shown here for varying  $F_1^p/F_2^p$ .

In Fig. 11 we plot the dynamic phase diagram for  $F^D$  versus  $F_1^p$  in a sample with equal pinning strength in both channels,  $F_2^p = F_1^p$ , and with  $d/a_1 = 1.7$ . For  $F_1^p < 0.7$ , the depinning is elastic and the depinning force  $F_c$  drops to zero as  $F_1^p$  drops to zero. For  $F_1^p > 0.8$ , the depinning is plastic and all of the dynamical transition lines shift to higher  $F^D$  for increasing  $F_1^p$ , with the S to L transition increasing the most rapidly. In the inset of Fig. 11 we plot  $dF_c/dF_1^p$  versus  $F_1^p$  where we find a roughly linear increase in the regime  $0 < F_1^p < 0.55$ . This implies that  $F_c \propto (F_1^p)^2$ , which is consistent with elastic depinning in the collective regime [14]. Just above the transition to plastic depinning,  $dF_c/dF_1^p$  passes through a strong drop and then recovers to a nearly constant value near 1, consistent with  $F_c \propto F^p$  as expected for the single particle limit. This results indicates that the transition from elastic to plastic depinning is associated with clearly observable features in the transport curves. If we fix  $F_1^p$  and instead increase the density of pinning sites while holding  $n_1 = n_2$ , we find dynamical phases with tendencies similar to those shown in Fig. 11.

## V. DENSITY DEPENDENCE AND DYNAMIC COMMENSURATION EFFECTS

We next consider the effect of holding the distance  $d$  between channels fixed while varying the density of the

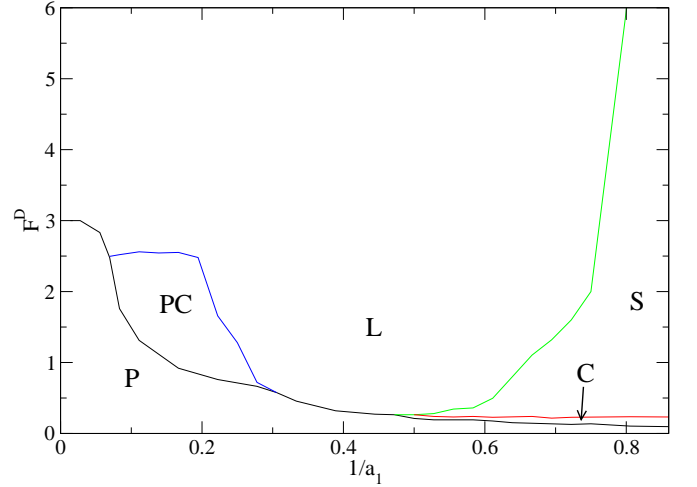


FIG. 12: Dynamic phase diagram  $F^D$  vs density  $1/a_1$  for an  $M = 2$  sample with  $N_1 = N_2$ ,  $n_1 = n_2$ ,  $a_1 = a_2$ ,  $n_1/N_1 = 0.25$ ,  $R_p = 0.35a_0$ ,  $d = 2.75a_0$ , and  $F_1^p = F_2^p = 3.0$ . P: pinned phase; C: coexistence phase; S: sliding phase; L: locked phase. For  $0.07 < 1/a_1 < 0.305$ , the system transitions from a pinned phase into a pulsed coexistence phase PC.

particles which changes the lattice constants  $a_1$  and  $a_2$ . In Fig. 12 we plot the dynamic phase diagram as a function of  $F^D$  and density  $1/a_1$  for varied  $a_1 = L/N_1$  in a system with an equal number of particles in each channel,  $N_1 = N_2$ ,  $F_1^p = F_2^p = 3.0$ , and with fixed  $d = 2.75$ . The depinning threshold  $F_c$  generally decreases with increasing density due to the increase in the particle-particle interactions relative to the pinning strength as the particles get closer together within each channel. For intermediate densities of  $0.305 < 1/a_1 < 0.47$ , the depinning is elastic, while for  $0.07 < 1/a_1 < 0.305$ , the depinning is plastic and the sample enters either the coexistence phase C, the sliding phase S, or a pulsed coexistence phase PC. Each channel is alternately pinned or moving in the PC phase, so that this phase resembles the C phase except that the stationary and moving channels keep switching places. At lower densities  $1/a_1 < 0.305$ , the system no longer depins elastically due to the increase of the effectiveness of the pinning, which induces a decoupling of the channels. For  $1/a_1 < 0.07$ , we have  $N_1 > n_1$ . Since there are more pinning sites than particles, the depinning occurs in the single particle limit and it is no longer meaningful to describe the pinning as plastic or elastic since the particles in the channels are essentially noninteracting at depinning. Additionally, since  $F_1^p$  and  $F_2^p$  are held fixed, at these low densities all the particles in each channel depin simultaneously and move at the same velocity so the C and S phases cannot occur.

Figure 12 shows that for  $1/a_1 > 0.47$ , the depinning is plastic again and the S-L transition shifts to higher  $F^D$  with increasing  $1/a_1$ . This might seem surprising since at higher densities the interactions between particles in the same channel become stronger; however, it is known

from studies of layered vortex systems that increasing the field can reduce the coupling *between* layers for fields well below  $H_{c2}$  [40]. As the density increases, the interactions between particles in the same channel become stronger more rapidly than the interactions between particles in neighboring channels. This enhances the S phase since it becomes more difficult for the channels to lock. In order to minimize energy, the moving particles adopt a zig zag configuration with the particles in one channel offset relative to the particles in the other channel. This approximates a triangular lattice configuration. As  $1/a_1$  increases, the lattice spacing along the channels shrinks but the lattice spacing perpendicular to the channels is fixed, producing an effectively increasingly anisotropic triangular lattice configuration. Eventually the spacing becomes so anisotropic that there is very little energy difference between an alternating configuration and one in which the particles are adjacent to each other, so the channels decouple.

The phase diagram in Fig. 12 exhibits a number of features found in phase diagrams for strongly layered superconductors containing disordered pinning sites. For example, the superconducting system undergoes a low field decoupling transition when the pancake vortices in each layer are far apart and only weakly interacting, while at higher fields there is a field induced decoupling [40]. In experiments, samples show both a low field disordering transition and a high field decoupling transition [44]. It was argued that the low field transition is pinning induced and occurs when the pinning energy overwhelms the weak vortex-vortex interactions at low vortex density, while the higher field disordering transition indicates the onset of the peak effect [44]. In our system, as we increase  $F^p$  the lower and upper endpoints of the elastic depinning regime approach each other until the elastic depinning disappears completely. Conversely as we decrease  $F^p$  the elastic depinning region becomes more extended. Although there is a small increase in  $F_c$  at the lower transition from plastic to elastic depinning, we find no feature in  $F_c$  at the upper transition from elastic to plastic depinning; however, if we measure the average particle velocity for fixed drive, the velocity drops across the higher density transition, similar to the effect illustrated in Fig. 4.

We next examine the effects of varied particle ratio for a sample with  $F_1^p = F_2^p = 6.0$ ,  $n_1 = n_2$ ,  $d/a_2 = 1.33$ , and  $n_2/N_2 = 0.167$ . We fix  $N_2$  and vary  $N_1$ . In Fig. 13(a) we plot  $2V_1/N$  and  $2V_2/N$  versus  $F^D$  for a sample with  $N_1/N_2 = 1/2$ . Here the velocities are normalized by  $N/2$ , where  $N = N_1 + N_2$ . In this case, channel 2 contains a larger number of particles and depins at lower  $F^D$  than channel 1. The slope of  $2V_2/N$  is also greater than that of  $2V_1/N$ . We replot the same curves in Fig. 13(b) with the normalizations  $V_1^* = V_1/N_1$  and  $V_2^* = V_2/N_2$ . Under this normalization the curves have almost the same form as the velocity force curves in samples with an equal number of particles in each channel, and it is now possible to distinguish the transition into a locked phase, which also

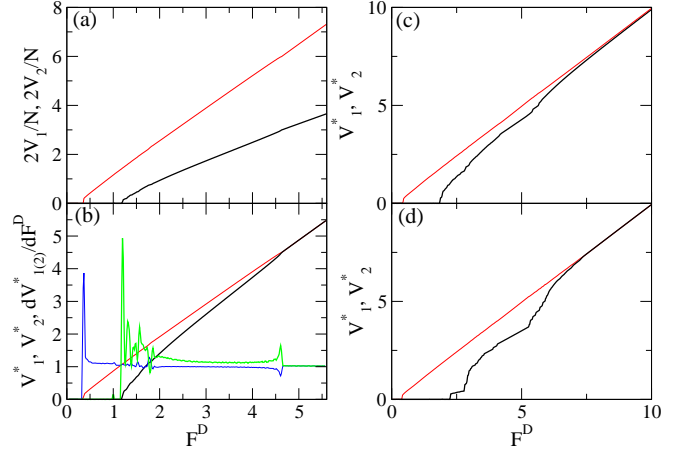


FIG. 13: Channel velocities vs  $F^D$  for an  $M = 2$  system with  $F_1^p = F_2^p = 6.0$ ,  $n_1 = n_2$ ,  $n_2/N_2 = 0.167$ ,  $R_p = 0.23a_0$ , and  $d/a_2 = 1.33$  for fixed  $N_2$  and varied  $N_1$ . (a)  $2V_1/N$  (lower dark line) and  $2V_2/N$  (upper light red line) for  $N_1/N_2 = 1/2$ . The velocities are normalized by  $N/2$ , where the total particle number  $N = N_1 + N_2$ . Channel 2 contains a larger number of particles than channel 1 and therefore  $V_2$  increases more rapidly than  $V_1$ . (b) The same curves plotted as  $V_1^* = V_1/N_1$  (lower dark line) and  $V_2^* = V_2/N_2$  (upper light red line). Here a clear dynamical locking transition appears. Also shown are the corresponding  $dV_1^*/dF^D$  (heavy green line) and  $dV_2^*/dF^D$  (light blue line) curves. (c)  $V_1^*$  (lower dark line) and  $V_2^*$  (upper light red line) for  $N_1/N_2 = 0.375$ . Here no dynamical locking occurs within this range of  $F^D$ . (d)  $V_1^*$  (lower dark line) and  $V_2^*$  (upper light red line) for  $N_1/N_2 = 0.292$  where dynamical locking occurs.

appears as a signature in the  $dV_1^*/dF^D$  and  $dV_2^*/dF^D$  curves shown in Fig. 13(b). At other fillings such as  $N_1/N_2 = 0.375$  shown in Fig. 13(c), there is no dynamical locking within this range of  $F^D$ . For lower fillings, dynamical locking appears again as shown in Fig. 13(d) for  $N_1/N_2 = 0.292$ . We find dynamical locking only for  $N_1/N_2 = 1.0$ ,  $N_1/N_2 = 1/2$ , and  $N_1/N_2 < 0.333$ .

Fig. 14(a) shows the dynamic phase diagram for  $F^D$  versus  $N_1/N_2$  for the system in Fig. 13. The solid line at  $N_1/N_2 = 1.0$  indicates that at this ratio the depinning is elastic and the system passes directly from the pinned to the moving locked phase. For  $N_1/N_2 < 1.0$ , channel 2 depins first since it contains a larger number of particles, and the depinning threshold of channel 1 grows with decreasing  $N_1/N_2$  until reaching a maximum value of  $F_c = F_1^p = 6.0$  at the lowest values of  $N_1/N_2$ . The dotted line at  $N_1/N_2 = 1/2$  in Fig. 14(a) indicates a transition from the S to the L phase at  $F^D = 4.5$  as illustrated in Fig. 13(b). For lower fillings  $N_1/N_2 \leq 0.33$  the system enters the locked state above  $F^D = 7.0$ . For  $N_1/N_2 > 1.0$ , channel 1 depins first since it now contains a larger number of particles than channel 2, and the depinning threshold for channel 2 remains nearly constant as  $N_1/N_2$  increases. We note that there could be a dynamical locking at external drives much higher than the

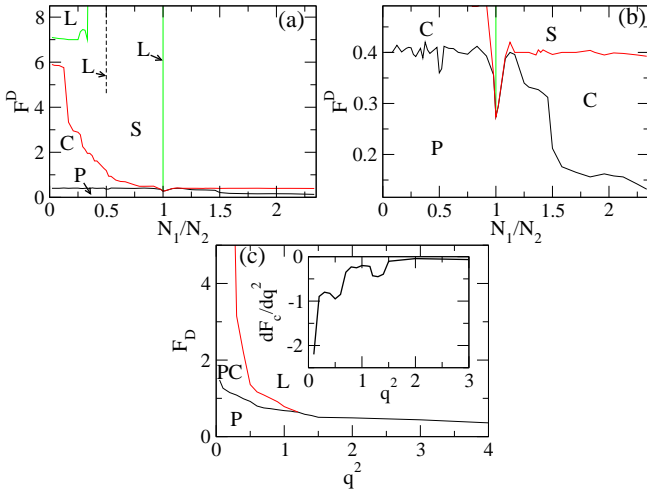


FIG. 14: (a) Dynamic phase diagram  $F^D$  vs  $N_1/N_2$  for the  $M = 2$  system in Fig. 13 with  $n_1 = n_2$ ,  $n_2/N_2 = 0.167$ ,  $R_p = 0.23a_0$ ,  $d/a_2 = 1.33$  and  $F_1^p = F_2^p = 6.0$ . P: pinned phase; C: coexistence phase; S: sliding phase; L: locked phase. The line at  $N_1/N_2 = 1.0$  indicates that for this filling only the system depins elastically directly into the L phase. The dashed line at  $N_1/N_2 = 0.5$  indicates a transition from the S phase into the L phase at this filling. (b) A blowup of panel (a) in the region near the depinning line shows dips in the depinning force  $F_c$  at  $N_1/N_2 = 0.5$  and  $1.0$ . (c) Dynamic phase diagram  $F^D$  vs  $q^2$ , where  $q^2$  is the dimensionless squared colloid charge, for a sample with  $n_1 = n_2$ ,  $n_2/N_2 = 0.167$ ,  $N_1/N_2 = 1$ ,  $R_p = 0.23a_0$ ,  $F_1^p = F_2^p = 6.0$ , and  $d/a_1 = 1.33$ . PC: the pulsed coexistence phase described in Fig. 12. For strong interactions (high  $q^2$ ) the depinning is elastic. For small  $q^2$  the depinning threshold is high. Inset:  $dF_c/dq^2$  vs  $q^2$  for the same sample shows that at  $q^2 = 1.15$  at the transition from plastic to elastic depinning there is a peak corresponding to a change in the slope of the depinning curve.

range of  $F^D$  we consider here.

In Fig. 14(b) we plot a blowup of the phase diagram from Fig. 14(a) near the depinning transition. Clear local minima in  $F_c$  appear at  $N_1/N_2 = 1.0$  where the depinning is elastic and at  $N_1/N_2 = 0.5$  where the depinning is not elastic but where the dynamical locking occurs at a much lower drive than for nearby fillings. The behavior at these two fillings can be viewed as a commensuration effect, where the coupling between the layers is enhanced at integer and certain fractional ratios of the filling factors. Commensuration effects have been studied in a variety of solid-on-solid systems where there is matching between the number of particles and the number of potential minima [45]. An example of such a system is vortices in type-II superconductors with periodic pinning sites where peaks in the critical depinning force occur at integer [46] and rational [47] matching fields. In our system commensuration occurs not between the number of particles and the number of pinning sites but between the number of particles in the different channels. The depinning force is reduced at the commensurate fillings due to the enhanced coupling between the layers. The

strongest dip in  $F_c$  occurs when the depinning is elastic at  $N_1/N_2 = 1.0$ , while for fillings just above and below this value  $F_c$  passes through local maxima. At the incommensurate fillings, there are geometrically necessary topological defects that enter the zig-zag structure formed by the particles in the two channels. These defects effectively soften the structure and reduce the coupling between the layers. This is similar to what occurs in the peak effect where the depinning force is higher for plastic depinning than for elastic depinning. We find no apparent anomaly at  $N_1/N_2 = 2.0$  in Fig. 14(b); however, we expect that for other parameter values, more commensuration effects would be observable and that additional dynamical locking regimes would also appear at other fillings. Commensuration effects are also observed in two layer and three layer systems without pinning when only one layer is driven [39]. In these systems the commensuration effects occur at fillings where the channels are more strongly coupled, so that the drive at which relative slip begins to occur between the channels is much higher than for incommensurate fillings. Our results indicate that a distinct commensuration effect can occur in layered systems that differs from commensuration effects observed for particles moving over fixed substrates.

It is also possible to change the overall particle-particle interaction strength by varying  $q^2$ . This can be achieved for colloidal systems by altering the effective charge on the particles or by changing the screening length. For vortex systems the vortex-vortex interactions can also change significantly due to thermal effects near  $T_c$ . In Fig. 14(c) we plot the dynamic phase diagram for  $F^D$  versus  $q^2$  for the system in Fig. 14(a) with  $N_1/N_2 = 1$ ,  $F_1^p = F_2^p = 6.0$  and  $d/a_2 = 1.33$ . For  $q^2 > 1.23$  or strong particle-particle repulsion the system depins elastically and as  $q^2$  increases  $F_c$  gradually decreases. Unlike the case for constant  $q^2$  but increasing density  $1/a_1$  shown in Fig. 12, we find in Fig. 14(c) that there is no elastic-plastic transition at high  $q^2$  since the zig-zag particle configuration is not affected by increasing  $q^2$ . When  $q^2$  is lowered,  $F_c$  increases and a transition from elastic to plastic depinning occurs at  $q^2 = 1.15$ . For  $q^2 < 1.15$ , the system depins into the pulsed coexistence (PC) phase described earlier in Fig. 12. The transition into the L phase also shifts to higher  $F^D$  as  $q^2$  decreases. The overall features of the phase diagram in Fig. 14(c) resemble those of the dynamic phase diagrams constructed for 2D vortex systems when the vortex-vortex interaction strength is varied [15, 48]. In the vortex case,  $F_c$  increases as the vortex-vortex interactions become weaker while the dynamical ordering transition shifts to higher values of the driving force. In the inset of Fig. 14(c) we plot  $dF_c/dq^2$  versus  $q^2$ , showing more clearly the change in the slope of  $F_c$  at the transition from plastic to elastic depinning at  $F^D = 1.15$ .

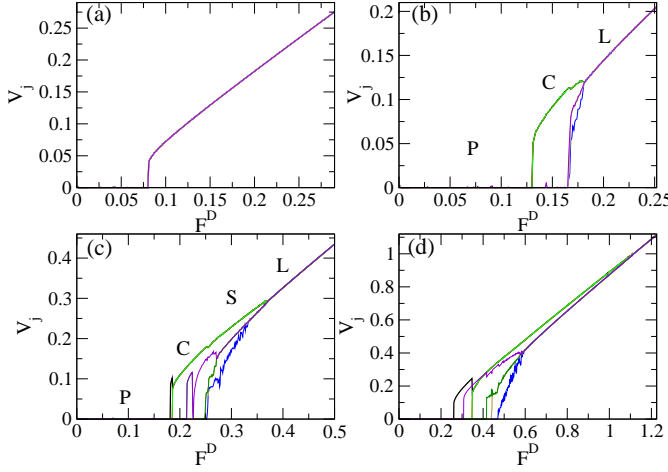


FIG. 15:  $V_j$  vs  $F^D$  for an  $M = 8$  system with  $N_j = N_1$ ,  $n_j = n_1$ ,  $n_1/N_1 = 0.17$ ,  $R_p = 0.23a_0$ ,  $a_1 = 1.5a_0$ , and  $d/a_1 = 1.33$ . (a) At  $F_j^p = F_1^p = 3.0$ , the depinning is elastic and all channels depin simultaneously into the locked state. (b) At  $F_j^p = F_1^p = 5.0$ , plastic depinning occurs and groups of channels lock together with each group depinning at a different value of  $F^D$ . At high drives all the channels lock. P: pinned phase; C: coexistence phase, defined as beginning when the first channel depins and ending when the final channel depins; L: locked phase. (c) At  $F_j^p = F_1^p = 7.0$ , more channels depin individually. A window of the sliding (S) phase appears where all channels are moving but groups of channels are separately locked. (d) At  $F_j^p = F_1^p = 11.0$ , the same four dynamical phases appear but are shifted to higher values of  $F^D$ .

## VI. MORE THAN TWO CHANNELS

We next consider the case of  $M > 2$  coupled driven channels. Figure 15 shows the velocity-force curves  $V_1$  through  $V_8$  versus  $F^D$  for an  $M = 8$  channel system with  $N_j = N_1$ ,  $n_j = n_1$ ,  $d/a_1 = 1.33$ , and  $n_1/N_1 = 0.17$ . Note that the boundary transverse to the channels is open and that there is therefore no significant direct interaction between the particles in channel 1 and those in channel 8. In Fig. 15(a) at  $F_j^p = F_1^p = 3.0$ , the depinning is elastic and the system immediately enters the locked phase upon depinning. All the channels depin simultaneously and the velocity curves overlap completely. In Fig. 15(b) at  $F_j^p = F_1^p = 5.0$  the depinning is plastic. Channels 1, 2, and 3 depin first and remain locked with each other, followed by the depinning of a group of four additional locked channels and then by the depinning of the final channel. At  $F^D = 0.182$  all the channels become dynamically locked together. For  $F_j^p = F_1^p = 7.0$  in Fig. 15(c), the system exhibits a hierarchy of dynamical locking transitions beginning at  $F^D = 0.186$  when the first three channels to depin become locked together. This is followed by a second dynamic locking of four different channels at  $F^D = 0.25$ . Channel 8 dynamically locks with these four channels near  $F^D = 0.335$ , and a

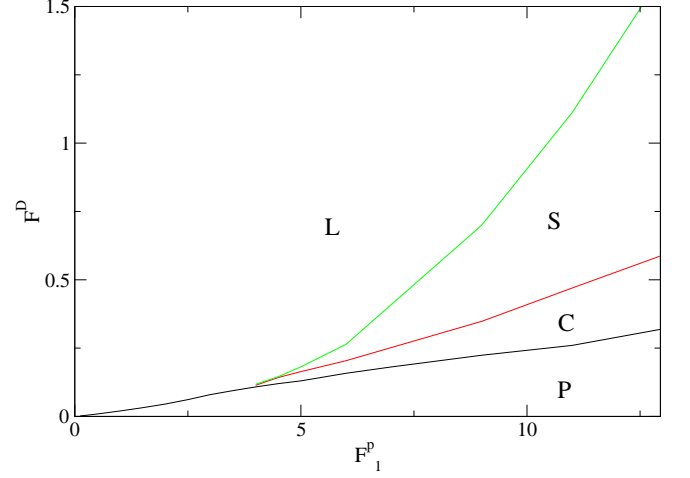


FIG. 16: Dynamic phase diagram  $F^D$  vs  $F_1^p$  for the  $M = 8$  system in Fig. 15 with  $N_j = N_1$ ,  $n_j = n_1$ ,  $F_j^p = F_1^p$ ,  $n_1/N_1 = 0.17$ ,  $R_p = 0.23a_0$ ,  $a_1 = 1.5a_0$ , and  $d/a_1 = 1.33$ . P: pinned phase; C: coexistence phase; S: sliding phase; L: locked phase.

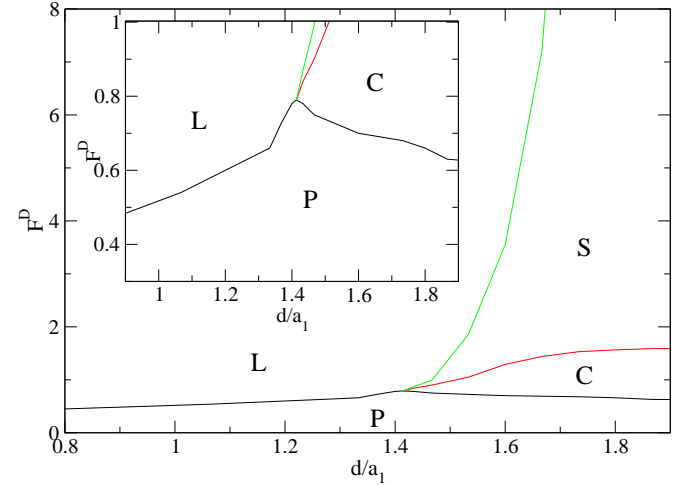


FIG. 17: Dynamic phase diagram  $F^D$  vs  $d/a_1$  for the  $M = 8$  system in Fig. 16 with  $F_j^p = F_1^p = 3.0$ ,  $N_j = N_1$ ,  $n_j = n_1$ ,  $n_1/N_1 = 0.17$ ,  $R_p = 0.23a_0$ , and fixed  $a_1 = 1.5a_0$ . P: pinned phase; C: coexistence phase; S: sliding phase; L: locked phase. Inset: Blow up of the region near the transition from elastic to plastic depinning shows that the depinning force  $F_c$  peaks at the transition.

final locking of all the channels with each other occurs at  $F^D = 0.37$ . As  $F^p$  increases the number of distinct dynamical locking transitions increases, as illustrated in Fig. 15(d) for  $F_j^p = F_1^p = 11.0$ , while the final transition at which all of the channels lock together shifts to higher values of  $F^D$ . Figure 15(d) also shows that it is possible for individual channels to exhibit negative differential conductivity at the onset of channel locking; however, there is generally no NDC in the summed velocity  $V = \sum_j^M V_j$ .

Figure 16 shows the dynamic phase diagram  $F^D$  versus



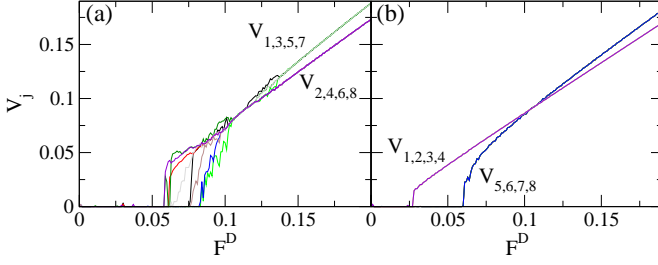


FIG. 18:  $V_j^*$  vs  $F^D$  for an  $M = 8$  system with  $N_j \neq N_1$ ,  $n_j = n_1$ ,  $F_j^p = F_1^p = 2.0$ ,  $n_j/N^{\text{high}} = 0.167$ ,  $R_p = 0.23a_0$ , and  $d = 1.7a_0$ . Each channel has one of two possible densities:  $d/a_j = 1.33$  for  $N_j = N^{\text{high}}$  or  $d/a_j = 1.2267$  for  $N_j = N^{\text{low}}$ . All of the velocities are normalized by  $V_j^* = V_j/N^{\text{high}}$ . (a) A sample with alternating density  $N_1 = N_3 = N_5 = N_7 = N^{\text{high}}$  and  $N_2 = N_4 = N_6 = N_8 = N^{\text{low}}$ . All eight channels depin at different values of  $F^D$  and undergo a series of locking and unlocking transitions until reaching a state at higher drives where all the low density channels are locked together and all the high density channels are locked together. (b) A sample with the same parameters but with segregated density  $N_1 = N_2 = N_3 = N_4 = N^{\text{low}}$  and  $N_5 = N_6 = N_7 = N_8 = N^{\text{high}}$  shows a two step depinning process.

$F_1^p$  for the  $M = 8$  system in Fig. 15 with  $F_j^p = F_1^p$ . The broad features of the phase diagram are the same as those found for the  $M = 2$  case in Fig. 11; however, there are multiple depinning transitions within the C phase and multiple locking transitions within the S phase that are not marked in Fig. 16. In Fig. 17 we plot the dynamic phase diagram  $F^D$  versus  $d/a_1$  for the same  $M = 8$  system with  $F_j^p = F_1^p = 3.0$  and fixed  $a_1 = 1.5a_0$ . At low  $d/a_1$  the system depins elastically, and a transition to plastic depinning occurs near  $d/a_1 = 1.4$ . For  $d/a_1 > 1.4$ , the transition between the S and L phases shifts to higher  $F^D$  with increasing  $d/a_1$ , while the general shape of the phase diagram resembles that of the  $M = 2$  system shown in Fig. 3. The C and S phases again contain multiple depinning and dynamical locking transitions, respectively. We observe a larger amount of hysteresis in  $M > 2$  systems than in the  $M = 2$  system. In the inset of Fig. 17 we plot a blowup of the  $F^D$  versus  $d/a_1$  phase diagram near the depinning transition to show that  $F_c$  passes through a peak at the transition from elastic to plastic depinning, similar to the peak found for the  $M = 2$  system in the inset of Fig. 3(a).

For systems in which the number of particles per channel is allowed to vary,  $N_j \neq N_1$ , the dynamic behavior becomes more complex; however, it is possible to identify several general features. In Fig. 18 we plot  $V_j^*$  versus  $F^D$  for an  $M = 8$  channel with  $F_j^p = F_1^p = 2.0$  and  $d = 1.7a_0$  where each channel contains either  $N^{\text{high}}$  particles with  $d/a_j = 1.33$  or  $N^{\text{low}}$  particles with  $d/a_j = 1.2267$ . The velocities are normalized by  $V_j^* = V_j/N^{\text{high}}$ , and the number of pins in each channel is  $n_j/N^{\text{high}} = 1.67$ . In samples where all channels have equal numbers of parti-

cles,  $N_j = N_1 = N^{\text{high}}$  or  $N_j = N_1 = N^{\text{low}}$ , the depinning is elastic for this set of parameters. The depinning becomes plastic when the number of particles differs from channel to channel. Figure 18(a) illustrates an alternating density system with  $N_1 = N_3 = N_5 = N_7 = N^{\text{high}}$  and  $N_2 = N_4 = N_6 = N_8 = N^{\text{low}}$ . Here every channel has a unique depinning threshold. There are multiple locking and unlocking transitions as channels become locked with a nearest neighbor channel at lower drives only to unlock at higher drives. The locking at lower drives occurs when the pinning is still able to create density inhomogeneities within individual channels. Particles form higher density regions in the areas behind the more effective pins where pileups occurred below the depinning transition. These higher density regions create an additional energy barrier for relative slip between the particles in neighboring channels, favoring the coupling of adjacent channels. This process creates a window  $0.105 < F^D < 0.115$  where all of the channels couple together,  $V_j = V_1$ . As  $F^D$  increases, the effectiveness of the pinning is reduced and the density within each channel becomes more homogeneous. At the same time, the overall effectiveness of the pinning in channels with  $N^{\text{high}}$  is reduced relative to that of the pinning in channels with  $N^{\text{low}}$  due to the stronger particle-particle interactions in the higher density channels. The velocity  $V^{\text{high}}$  in channels with  $N^{\text{high}}$  is therefore higher than the velocity  $V^{\text{low}}$  in channels with  $N^{\text{low}}$ ,  $V^{\text{high}} > V^{\text{low}}$ , which favors decoupling of adjacent channels in the alternating density system. For high enough drives  $F^D > 0.14$ , the system enters a doubly locked phase where all channels containing the same number of particles are locked together,  $V_1 = V_3 = V_5 = V_7$  and  $V_2 = V_4 = V_6 = V_8$ , but  $V_1 > V_2$  by a small amount so that the two sets of locked channels slip with respect to each other.

There are many other possible ways to arrange the  $M = 8$  channels such that half of the channels contain  $N^{\text{high}}$  particles and half of the channels contain  $N^{\text{low}}$  particles. For example, Fig. 18(b) shows  $V_j^*$  versus  $F^D$  in a density segregated system with  $N_1 = N_2 = N_3 = N_4 = N^{\text{low}}$  and  $N_5 = N_6 = N_7 = N_8 = N^{\text{high}}$ . Here each group of channels with equal density acts as a unit and depins elastically into a locked moving state; however, the two groups of channels do not lock with each other. The depinning occurs in two steps with a lower depinning threshold for the set of channels containing  $N^{\text{low}}$  particles. This behavior resembles that of an  $N = 2$  sample with  $N_1/N_2$  slightly below 1, as in Fig. 14(b), which depins into the coexistence phase and then enters the sliding phase at higher  $F^D$ . In Fig. 18(b) we find that  $V_1^* > V_5^*$  just above the second depinning transition but  $V_1^* < V_5^*$  at high  $F^D$ , with a crossing of the velocity curves occurring near  $F^D = 0.105$ .

In samples with an equal number of particles per channel, particles in neighboring channels can organize into a two-dimensionally ordered state in order to minimize the particle-particle interaction energy. This ordered configuration can be perturbed by changing the number of



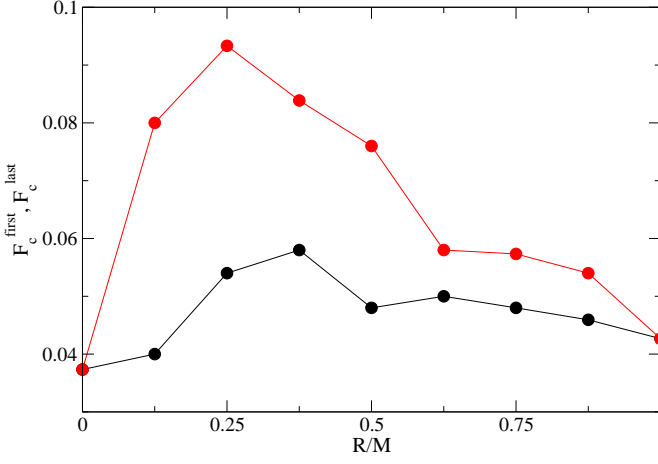


FIG. 19: The depinning thresholds  $F_c^{\text{first}}$  (lower curve) and  $F_c^{\text{last}}$  (upper curve) vs  $R/M$  for the  $M = 8$  sample from Fig. 18(a) with  $n_j = n_1$ ,  $F_j^p = F_1^p = 2.0$ ,  $n_j/N^{\text{high}} = 0.167$ ,  $R_p = 0.23a_0$ ,  $d = 1.7a_0$ ,  $d/a_j = 1.133$  for  $N_j = N^{\text{low}}$ , and  $d/a_j = 1.216$  for  $N_j = N^{\text{high}}$ . There are  $R$  evenly spaced channels with  $N_j = N^{\text{high}}$  and the remaining  $M - R$  channels have  $N_j = N^{\text{low}}$ . When the system is commensurate with  $N_j = N_1$  at  $R/M = 0$  or  $R/M = 1$ , the depinning is elastic. For other values of  $R/M$  the depinning is plastic, and the maximum depinning thresholds appear when geometrically necessary dislocations form in the system.

particles in one or more channels, which causes defects to appear. To study this, we consider an  $M = 8$  system in which all channels initially contain  $N_j = N^{\text{low}}$  particles with  $d/a_j = 1.133$ . We select  $R$  channels evenly spaced across the system with  $0 \leq R \leq M$  and increase the number of particles in the selected channels to  $N_j = N^{\text{high}}$  with  $d/a_j = 1.216$ . As  $R$  varies, the system passes from a state in which all channels have  $N_j = N^{\text{low}}$  at  $R/M = 0$  to one in which all channels have  $N_j = N^{\text{high}}$  at  $R/M = 1$ . Fig. 19 shows  $F_c^{\text{first}}$ , the value of  $F^D$  at which the first channel depins, and  $F_c^{\text{last}}$ , the value of  $F^D$  at which the final channel depins, for different values of  $R/M$ . The geometrically necessary dislocations that form for intermediate values of  $R/M$  are aligned with the direction of the drive. At  $R/M = 0$  and 1, dislocations are absent and the depinning transition is elastic, but for other values of  $R/M$  when dislocations are present, the depinning is plastic and the depinning thresholds are shifted to higher values of  $F^D$ . This can also be seen in Fig. 18: the alternating filling sample in Fig. 18(a) contains numerous topological defects and has much higher depinning thresholds than the segregated filling sample in Fig. 18(b), where there are only dislocations along the interface between the two fillings. This result again indicates that in our model the higher depinning forces are associated with plastic depinning where the channels can slide past each other due to the presence of topological defects created either by geometrical necessity for channels with unequal particle numbers or by the quenched

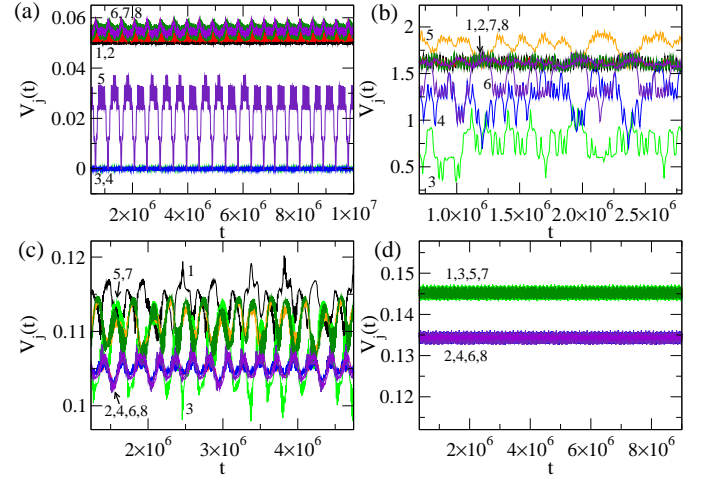


FIG. 20: The time trace of the velocities of the individual channels,  $V_j(t)$ , for the  $M = 8$  sample from Fig. 18(a) with  $N_1 = N_3 = N_5 = N_7 = N^{\text{high}}$  and  $N_2 = N_4 = N_6 = N_8 = N^{\text{low}}$ . (a) At  $F^D = 0.08$  the system is in the coexistence phase. Channels 3 and 4 are pinned, channel 5 flows intermittently, channels 1 and 2 are locked at a low average velocity, and channels 6, 7, and 8 are locked at a higher average velocity. (b) At  $F^D = 0.09$  there is considerably more intermittency. Here channels 1, 2, 7 and 8 are locked continuously, while channels 3, 4, 5, and 6 show intermittent locking with the other channels. (c) At  $F^D = 0.125$  there is less intermittency and a larger number of the channels are locked with other channels. (d) At  $F^D = 0.16$ , channels 1, 3, 5, and 7 are locked and due to their higher density move with a higher velocity than the remaining channels 2, 4, 6, and 8, which are locked together.

disorder. The results in Fig. 18 and Fig. 19 are similar to previous work on mixtures of particles in the presence of a weak disordered substrate, which showed that the depinning force is minimized when only one particle species is present and there are no topological defects, but that for intermediate mixtures of particle species, topological defects appear and increase the depinning force [49].

The locking-unlocking transition that occurs for the alternating density sample in Fig. 18(a) can be better characterized by analyzing the time series  $V_j(t)$  of the velocities of the individual channels at different values of  $F^D$ . In Fig. 20(a), where we plot  $V_j(t)$  at  $F^D = 0.08$ , the system is in the coexistence regime and channels 3 and 4 are pinned. Channel 5 switches between a temporarily pinned state and a sliding state, with occasional jumps to higher velocity flow. Channels 1 and 2 are locked together and have an average velocity of  $V_1 = V_2 = 0.052$ , while channels 6, 7, and 8 are locked together at a slightly higher average velocity of  $V_6 = V_7 = V_8 = 0.055$  and show a pronounced modulation at a frequency which matches the frequency at which  $V_5$  drops to zero. The point at which  $V_5$  reaches zero corresponds to the point at which  $V_6$ ,  $V_7$ , and  $V_8$  reach their maximum values. Numerous periodic oscillations appear in the velocities,

such as the washboard frequency of the particles moving over the pinning sites. Additionally, each channel has a periodicity determined by the value of  $N_j$ , and since this frequency differs from channel to channel, the channels sliding past one another experience a dynamic periodic potential caused by the periodicity of the particles. This slipping process is not completely periodic but shows some changes over time. For lower drives, more channels are pinned and the velocity signals become increasingly periodic. We note that for the  $M = 2$  case illustrated in Fig. 7 the velocity time traces were dominated by the fundamental frequencies which were washboard signals.

Figure 20(b) shows  $V_j(t)$  in the same sample for  $F^D = 0.09$ . Here there is considerable intermittency and several channels lock and unlock. Channels 1, 2, 7, and 8 are locked and move at  $V_1 = V_2 = V_7 = V_8 = 0.0625$ . Channel 3 has the lowest average velocity and intermittently locks with channel 4, which has the second lowest average velocity. Channel 4 locks intermittently with both channels 3 and 6, as well as much more infrequently locking with the group of channels 1, 2, 7, and 8. Channel 6 also locks intermittently with the channel group 1, 2, 7, and 8 as well as with channel 4. Channel 5 has the highest average velocity and intermittently locks to the lower velocity value of the group of channels 1, 2, 7, and 8. In Fig. 20(c) at  $F^D = 0.125$ , the amount of intermittency is reduced. Channels 2, 4, 6, and 8 are locked together and have a lower average velocity, while channels 5 and 7 lock together and have a higher but strongly fluctuating velocity. Channel 1, which moves independently, reaches the highest velocity values, while channel 3, which also moves independently, reaches the lowest velocity values. At  $F^D = 0.16$ , shown in Fig. 20(d), the high density channels 1, 3, 5, and 7 are locked together and move at a higher average velocity while the low density channels 2, 4, 6, and 8 also lock with each other and move at a lower average velocity.

The multiple channel system exhibits many of the features predicted in mean field studies of phase slip systems [37], such as the coexistence of sliding and pinned phases. There are, however, many features of the multiple channel system that are not captured by the mean field results, such as the multiple coupling and decoupling transitions between pairs of channels or groups of channels or the peak effect in the depinning threshold at the transition from elastic to plastic depinning. Additionally, for both the two channel and multiple channel systems, the case where different amounts of disorder exist in each channel has not been studied with mean field models. We note that there are many more parameters that can be explored for the multiple channel systems, such as having different densities or pinning strengths for each channel; however, the results for the dynamic phase diagram for the multiple channel system suggest that many of the generic features observed for the two level system will persist in the multiple channel system. We will explore this in more detail elsewhere.

## VII. SUMMARY

We have examined systems of two or multiple one-dimensional channels of coupled particles that interact repulsively within each channel and between the channels. The particles are uniformly driven with an external drive and in the presence of quenched disorder show a series of dynamic phases including a pinned phase, a coexistence between pinned and sliding phases, a sliding phase where the channels move at different average velocities and slip past one another, and a dynamically locked phase where the particle positions in the channels become locked and the channels move at the same velocity. The transition between these different phases can be observed as clear features in the velocity force curve characteristics. These features include a sudden drop in the velocity or the onset of negative differential conductivity in one of more of the channels at the dynamically induced locking transition. For weak pinning or strong channel coupling, the depinning occurs elastically without any slipping between channels. When the system parameters are varied, such as by decreasing the coupling between channels, we find a transition to plastic depinning, with the initial flow above depinning in the coexistence regime where the channels depin separately. At the transition between elastic and plastic depinning, a peak in the depinning force appears which resembles the peak effect phenomenon found in more complicated models with transitions between elastic and plastic depinning. We have also examined channels containing unequal numbers of pinning sites or particles and find that even for a channel containing no pinning, there can be a finite depinning threshold due to the coupling of the particles in the pin-free channel with the particles in a channel containing pinning. For unequal numbers of particles in the channels we observe commensurability effects where the depinning threshold drops when the depinning becomes elastic for particle number ratios at which the coupling between particles in neighboring channels is enhanced, such as at 1:1 or 2:1. At the incommensurate fillings the channels always depin plastically. For multiple channels we find a hierarchy of dynamically induced coupling phases in which different groups of channels couple, while at higher drive all of the channels become coupled. Despite the apparent simplicity of our model, we find that even the two channel system exhibits many of the prominent features of depinning in more complex models such as the existence of multiple dynamical phases as well as elastic to plastic depinning transitions of the type that have been studied for more complicated systems including 3D layered systems.

The specific model studied in this paper could be realized using colloidal particles in coupled 1D channels or in superconductors with a one-dimensional corrugation combined with random pinning where the vortices move along the easy flow direction of the corrugation. Other possible realizations include coupled channels of 1D wires where Wigner crystallization may occur.

## VIII. ACKNOWLEDGMENTS

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